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The Mechanical Behaviour of Polymers under High Pressure

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ABSTRACT

The effect of pressure on the tensile deformation of amorphous polycarbonate and poly(ethylene terephthalate) and semi-crystalline polychlorotrifluoroethylene and polytetrafluoroethylene was investigated up to 8 kb. Tensile deformations of polycarbonate at atmospheric pressure at temperatures down to 116° k were also performed. The former three polymers showed increases of yield stress, yield strain and elastic modulus, and decreases of fracture strain. Polytetrafluoroethylene behaved in an analogous manner up to 4 kb, beyond which both the ' yield ' stress and elastic modulus deviated systematically from the lower-pressure behaviour. This was found to correlate with changes of bulk modulus at a solid–solid phase transition near 5 kb.

The pressure dependence of yield stress was fitted by modified von Mises and Mohr–Coulomb yield criteria. A material parameter describing this pressure dependence was obtained for these and other polymers and was found to correlate qualitatively with the strength-limiting temperature, T_g or T_m , polytetrafluoroethylene being an exception. It was found that the pressure dependence of yield strain in polycarbonate could be accounted for by free volume considerations.

Correlation of high-pressure behaviour with low-temperature behaviour led to two conclusions: firstly, from a knowledge of bulk physical properties and a pseudothermodynamic equation, developed here, changes in lowtemperature yielding phenomena can be used to predict similar changes with high pressure, and, secondly, the ductile-brittle transition of a normally ductile amorphous polymer may be closely related to a specific dynamic mechanical relaxation, the temperature of observation changing with pressure.

§ 1. INTRODUCTION

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HYDROSTATIC pressure has been shown to have a significant effect on the mechanical properties of polymers. Holliday, Mann, Pogany, Pugh and Gunn (1964) have shown that polystyrene, which normally fractures in a brittle manner in tension, can be made to undergo yielding at considerably greater stresses if tested under hydrostatic pressures of 7 kb. Ainbinder, Laka and Maiors (1964) demonstrated that the yield maximum of a ductile polymer in nominal uniaxial compression was increased 150% by application of 2 kb hydrostatic pressure. The elastic modulus of a number of elastomers has been found to increase over a range of pressures

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from values characteristic of the rubbery state to those characteristic of the glassy state, and Paterson (1964) proposed that this was evidence for a pressure-shifted glass transition, the atmospheric-pressure glass transitions occurring at temperatures tens of degrees below the room temperature testing conditions.

Semi-crystalline polymers were studied by Ainbinder, Laka and Maiors (1965) and Laka and Dzenis (1967), though only to 2 kb. There were significant increases of strength under pressure for these also, and stress-whitening was eliminated in several cases. Sardar, Radeliffe and Baer (1968), utilizing an apparatus which enabled observation of the semicrystalline polyoxymethylene at pressures to 8 kb, showed that not only did the strength increase markedly, but also the mode and rate of fracture changed. The trends of several properties with pressure changed between 5 and 8 kb, and after comparisons with property changes at low temperatures it was proposed that in this pressure range the normally low-temperature β -relaxation (-75° c) had been shifted to room temperature. Changes in the large-strain deformation behaviour of two other semi-crystalline polymers (Mears, Pae and Sauer 1969) may also involve the shifting of low-temperature relaxations, particularly when related to ductile-brittle transitions.

More extensive experiments have been published on normally brittle polystyrene (Holliday and Mann 1968, Biglione, Baer and Radcliffe 1969) including optical observations of its deformation behaviour. These showed a brittle to ductile transition between 2 and 3 kb, attributed to changes in the critical crack size for Griffith flaws (Biglione *et al.* 1969). In the latter study a rubber-modified polystyrene was no longer ductile at 1 kb, supposedly due to suppression of crazing between rubber particles, and thereafter behaved as a ' diluted ' polystyrene.

The purpose of the present study was to extend high-pressure studies with optical observations to the ductile amorphous class of solid polymers. This is a class which has been the most extensively treated by mechanistic theories of yielding because of ductility below their glass transitions, the onset of ductility being associated with low-temperature mechanical relaxations which may be amenable to pressure-shifting.

§ 2. EXPERIMENTAL

2.1. Specimens

Four types of polymers were investigated, two ductile amorphous materials, additive-free molded $\frac{1}{2}$ in. square bars of poly (bisphenol A carbonate)[†] with $M_w = 35\,000$ to $36\,000$ and 93 mil sheets of poly (ethylene terephthalate)[‡], and two semi-crystalline polymers, commercial

[†] Merlon (R) polycarbonate from the Mobay Chemical Company.

[‡] From E. I. du Pont de Nemours & Co., Inc.

453

 $\frac{1}{2}$ in. diameter polychlorotrifluoroethylene[†] rods and ram-extruded $\frac{1}{2}$ in. diameter rods of polytetrafluoroethylene[‡]. In the text the names for these four polymers will often be abbreviated, respectively to PBAC, PET, PCTF, and PTFE.

Materials obtained as rods were machined to cylindrical shapes whose gauge dimensions were 0.150 in. diameter and 0.60 in. length. The PET sheet material was machined to gauge dimensions of 0.60 in. length, 0.20 in. width, and thickness as received.

2.2. Apparatus and Procedures

The specimens were strained in uniaxial tension in a chamber at confining pressures up to 8 kb using castor oil as the pressure-transmitting fluid. The high-pressure apparatus has an internal, pressure-compensating load cell and windows for viewing and photographing the specimen during the test; afterward the films are measured for determining true stresses and strains. The apparatus is a modified version (Das and Radcliffe 1968) of that described by Pugh and Green (1956) and Pugh, Lees, Ashcroft and Gunn (1961). The tests were conducted at constant crosshead velocity of 0.006 in. min⁻¹ at room temperature, $300^{\circ}\kappa$.

Polycarbonate specimens were also tested at low temperatures under atmospheric pressure in a cryostat using fluids to cool protected specimens by air convection and some conduction. The fluids at their use temperatures were water $(273^{\circ}\kappa)$, tetrachloroethylene $(251-253^{\circ}\kappa)$ monochlorobenzene $(229^{\circ}\kappa)$, solid carbon dioxide $(194^{\circ}\kappa)$ in an acetone bath, ethyl alcohol $(148^{\circ}\kappa)$ and isopentane $(116^{\circ}\kappa)$. These tests were also tensile in nature, the crosshead velocity being 0.005 in. min⁻¹. Further details can be found in the thesis of Christiansen (1970).

§ 3. Results

3.1. Experimental

The nominal stress-strain curves for the four polymers at various pressures are shown in figs. 1 to 4. For PBAC, PET, and PCTFE there were increase of modulus, yield stress and strain, and decreases of fracture strain. When the nominal stresses are corrected for the change of diameter the true stresses are obtained. From Considere constructions (see, for example, Vincent 1960) on true stress-strain plots the true yield stresses were obtained, and these increase linearly with pressure for PBAC, PCTFE, and PTFE up to 4 kb. The respective values of true yield stress at atmospheric pressure were 9000 p.s.i., 5700 p.s.i., and 850 p.s.i., respectively, for the last-mentioned three materials, and the slopes of their pressure dependence (in units of p.s.i./p.s.i.) were 0.145, 0.231 and 0.095.

[†]Kel-F^r 81, Grade III, trademark of the 3M Co.

[†] Teflon^r 6121 from E. I. du Pont de Nemours & Co., Inc.



Fig. 1







Engineering tensile stress-strain curves for poly(ethylene terephthalate) at various pressures.

Mechanical Behaviour of Polymers under High Pressure

The low-temperature nominal stress–strain curves for polycarbonate are presented in fig. 5. Again there was a general increase of modulus, yield stress and strain, and a decrease of fracture strain, which was quite drastically reduced in the small region between $273^{\circ}\kappa$ and $253^{\circ}\kappa$, unlike the high-pressure results.



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Engineering tensile stress–strain curves for polychlorotrifluorethylene at various pressures.

3.2.1. Yield criteria

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Applied mathematical formulations using relevant parameters have been constructed to predict the failure in any definable stress state. These rely on the condition that when a particular parameter reaches some 'critical' value by application of a system of stresses, the specimen will fail. For yielding, the most useful parameters are the maximum and octahedral shear stresses, τ_{max} and τ_{oct} , respectively. If the critical value of this

[†]The magnitude of the maximum shear stress τ_{\max} is calculated as $(\sigma_1 - \sigma_3)/2$, where σ_1 and σ_3 are the algebraically largest and smallest principal normal stresses, respectively. The octahedral shear stress is equal to $[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}/3$.



Engineering tensile stress–strain curves for polytetrafluorethylene at various pressures.

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Engineering tensile stress-strain curves for polycarbonate at various temperatures.

457

property is a constant regardless of the stress system, then the yield criteria are those of Tresca and von Mises, respectively. This is generally the case for ductile metals. When τ_{\max} is linearly dependent on the sum of the major and minor principal normal stresses $(\sigma_1 + \sigma_3)$ then this is referred to as the Mohr–Coulomb or Guest criterion.

In every relevant study to date, polymers have been found to exhibit significant increases of yield stress with pressure. Thus a yield criterion for polymers must include provision for the effect of pressure or mean normal stress, $\sigma_{\rm m} = (\sigma_1 + \sigma_2 + \sigma_3)/3$, on the magnitude of the critical shear stress. This would then result in either a slightly modified Mohr–Coulomb criterion, in the form

$$\tau_{\rm max} = \tau_0 - \mu' \sigma_{\rm m}$$

or a modified von Mises criterion in the form

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$$\tau_{\rm oct} = \tau_{\rm s} - \mu \sigma_{\rm m},$$

where τ_0 and τ_s are the atmospheric pressure pure shear maximum and octahedral yield stresses, and μ' and μ are materials parameters describing the pressure dependency of the respective yield stresses. Whitney and Andrews (1967) and Bowden and Jukes (1968) have applied the former equation to uniaxial and biaxial stress experiments. The latter equation was proposed by Sternstein and Ongchin (1969) after fitting biaxial stress data on poly (methyl methacrylate). Graphical analysis of these criteria for biaxial stress states shows that the modified Mohr–Coulomb criterion is represented by a distorted hexagon and the modified von Mises criterion by a distorted ellipse, the distortion occurring along the $\sigma_1 = \sigma_2$ direction and tending to foreshorten in the biaxial tensile direction and elongate in the biaxial compression direction. A modified von Mises criterion ellipse could just as well have been selected to fit the original data of Whitney (1964).

In the present case, where only uniaxial tension experiments were performed, no discrimination between the two is possible, since they are then mathematically related as $\tau_{\rm s} = \tau_0 (2\sqrt{2})/3$ and $\mu = \mu'(2\sqrt{2})/3$. For engineering use the differences between the two are insignificant, the modified von Mises criteria providing a slightly more conservative prediction.

For the application of yield criteria, true stress data should be used. When this was not possible, as with PET, estimates were made using engineering equations. Assuming a value for a Poisson's ratio (defined as the ratio of the transverse to axial nominal strains) independent of pressure, the true and nominal stresses are related as

where σ is the true stress, S is the nominal stress, ν has the value of the assumed Poisson's ratio, and e_1 is the nominal axial strain.

Using experimentally determined values of true yield stress where possible, and estimates of the true stress at yield from eqn. (1), in other cases, it was possible to calculate the points for plots of τ_{oct} versus σ_m for the materials used in the present study, fig. 6, and for those investigated previously. The slopes, μ , and the intercepts at $\sigma_m = 0$, τ_s , describing the relationships for the materials which exhibited linear behaviour are collected in table 1. Attempts to relate μ to values of bulk modulus, or to the pressure-dependence of the bulk modulus, were fruitless. However, an ordering by magnitude of the strength-limiting temperature, in separate categories of crystalline and amorphous, agreed qualitatively to a corresponding ranking by magnitude of μ , PTFE providing an exception in this case. The parameter μ may vary with crystallinity and other factors which affect yielding, and thus μ would not be expected to correlate with either T_g or T_m directly.



Yielding condition for four polymers plotted according to the Sternstein–Onchin (1969) shear yielding criterion.

3.2.2. Pressure-temperature correlation of yield stresses

Ainbinder (1969) formulated a relationship between changes of failure stress and changes of volume, such that for a specific volume change, caused by either a change of pressure or temperature, there will be a specific change of yield stress. That this is not generally true can be be shown with the high-pressure and low-temperature data obtained in this study plus dilatometric data from the literature, at high pressure (Warfield 1967) and low temperature (Hellwege, Hennig and Knappe 1962). As temperature is decreased from a reference condition there will be a change of volume and a simultaneous change of yield stress. This would likewise be true if the pressure were raised. If we plot the observed yield stresses against a common volume change abscissa (additional abscissa axes being incorporated to indicate the particular temperature or pressure) a graph such as fig. 7 results, from which it can be seen that there is not a unique

Mechanical Behaviour of Polymers under High Pressure 459

Material	μ	$(\mathrm{p.s.i.} \overset{\boldsymbol{\tau}_{\mathrm{s}}}{\times} 10^{-3})$	$T_{\mathrm{m}}^{\dagger}^{\dagger}_{(^{\circ}\mathrm{C})}$	$T_{ m g}$ (°C)
	Cry	ystalline		
PCTFE	0.12	2.9	220	
POM	0.10	7.3	181	
PPt	0.092	$3 \cdot 2$	165	
PEİ	0.046	2.0	137	
PTFE (to 4 kb)	0.048	0.4	327	
PTFE §	0.032	0.8	327	
	Am	norphous		
PBAC	0.072	4.5		150
PBAC	0.047	6.2		150
PS	0.084	8.3		100
PS¶	0.055	7.5		100
PET	0.054	4.2		69

Table 1. Values of parameters from the yield criterion $\tau_{oct} = \tau_s - \mu \sigma_m$

[†] Values taken from Nielsen (1962).

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t Calculated from the data of Mears et al. (1969).

§ Calculated from the data of Pae and Mears (1968). Calculated from the data of Mears and Pae (1969).

¶ Calculated from the data of Holliday and Mann (1968).



Fig. 7

Cross-plot of engineering yield stresses against volume changes at the corresponding temperatures or pressures.

relationship between volume change and change of yield stress. At any specified volume change, the effects of a changed temperature affect the yield stress more than do those of a changed pressure, thus negating Ainbinder's simple relationship.

There is, however, a similarity in the forms of the relations between yield stress and volume change which might allow a shifting to superimpose the behaviour if the proper factor is used. From fig. 7 it appears that some relatively constant proportion of the volume changes may provide this factor. Indeed, by going back to the defining equations for changes of volume by temperature and pressure, $\alpha = V_{\rm T}^{-1} (dV_{\rm T}/dT)_{\rm p}$ and $\beta = K^{-1} = V_{\rm p}^{-1} (dV_{\rm p}/dP)_{\rm T}$, and approximating these by differences, $\alpha \simeq V_{\rm T}^{-1} (\Delta V_{\rm T}/\Delta T)$ and $\beta \simeq K^{-1} = V_{\rm p}^{-1} (\Delta V_{\rm p}/\Delta P)$, one can obtain

$$(\alpha K)^{-1} \simeq (V_{\rm T}/V_{\rm p})(\Delta V_{\rm p}/\Delta V_{\rm T})(\Delta T/\Delta P).$$
 (2)

Since the ratio of volume changes is much further from unity than the ratio of volumes, the latter can be neglected to simplify the equation, which when rearranged can be written as

$$(\Delta T/\Delta P) \simeq (1/\alpha K) (\Delta V_{\rm T}/\Delta V_{\rm p}). \qquad (3)$$

Using both present results and literature values for changes of yield stress and volume with temperature and pressure, and for bulk modulus and thermal expansion, eqn. (3) can be evaluated. In table 2 the second and last numerical columns correspond to the left and right sides of eqn. (3) and show fairly good agreement, considering the diverse sources and

Material	Stress range (10 ³ p.s.i.)	$-\Delta T/\Delta P$ (°c $ imes 10^3$ /p.s.i.)	$(1/\alpha K)$ (°c × 10 ³ /p.s.i.)	$(\Delta V_{\mathrm{T}}/\Delta V_{\mathrm{p}})$	$(1/lpha K)(\Delta V_{ m T}/\Delta V_{ m p})$ (°c × 10 ³ /p.s.i.)
POM	4.0	$3 \cdot 4(a-d)$	$7 \cdot 6(a, e)$	0.67	5.1
PP	5.8	2.7(f-h)	$5 \cdot 1(h)$	0.54	2.8
PE	2.5	$2 \cdot 3(f, i-k)$	6.5(k, l)	0.53	3.4
PCTFE	12.0	$2 \cdot 3(b, l, m)$	$5 \cdot 5(b, n)$	0.38	2.1
PC	4.5	$2 \cdot 2(b, e)$	$8\cdot 3(e, o)$	0.28	2.3
PET	2.9	$1 \cdot 6(l, p, q)$	$5 \cdot 9(e, l)$	0.26	1.5
PS	9.1	$1 \cdot 2(b, k, r, s)$	$7 \cdot 5(l, t)$	0.26	1.9

Table 2.	Evaluation	of	equation	$-\Delta T$	$ \Delta P \simeq $	$(1/\alpha)$	K)	$(\Delta V_{\rm T})$	$ \Delta V_{\rm p} $)
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(a) Sardar et al. (1968); (b) Warfield (1967); (c) duPont; (d) Stehling and Mandelkern (1969); (e) Hellwege et al. (1962); (f) Mears, et al. (1969); (g) Vincent (1963 b); (h) Passaglia and Martin (1964); (i) Vincent (1963 a); (j) Warfield (1966); (k) Zakin, Simha and Hershey (1966); (l) Hellwege, Knappe and Lehmann (1962); (m) Mowers (1961); (n) Mowers 1962); (o) Hennig (1965); (p) Vincent (1963 c); (q) Haldon, Schell and Simha (1967); (r) Biglione et al. (1969); (s) Argon, Andrews, Godrick and Whitney (1968); (t) DiBenedetto (1963).

experimental details entailed. Thus the volume correction of eqn. (3) results in better agreement than Ainbinder's (1969) simpler relationship for superposing the effect of pressure and temperature with respect to yield stress.

3.2.3. Free volume and yield strains

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Free volume theories require a net dilational component to explain yielding. Such a formulation for tensile tests was proposed by Litt, Koch and Tobolsky (1967) and simplified by Rusch and Beck (1969) to a form

where $e_{\rm v}$ is the yield strain, $\alpha_{\rm g}$ and $\alpha_{\rm c}$ are the coefficients of thermal expansion for the glassy and crystalline states, respectively, T^* is the temperature at which the molecular relaxation processes occur at the same rate as that at which the polymer is being strained, and T is the test temperature. The variables $\alpha_{g}, \alpha_{c}, \nu$ and T^{*} could be expected to change with pressure, thus changing e_v . T* is closely associated with T_o , 160°c compared to 145°c at atmospheric pressure for polycarbonate, using the values of Litt et al. (1967) and so might vary with pressure in the same manner. The volumetric data of Matsuoka and Ishida (1966) indicate an increase of T_g by about 50° between atmospheric pressure and 1 kb for polycarbonate. Since the values of α_{g} and α_{e} are of the same order (Bondi 1968) one might expect their dependence on pressure to be about equal, thus negating the effect of pressure on their differences; lacking appropriate data on the crystalline state, this assumption will be made. Gielessen and Koppelmann (1960) found only a three per cent increase of ν up to 1 kb for PMMA, an increase which will be assumed here using an initial value of $\nu = 0.35$. Using these assumptions and a test temperature of 25° c the equation predicts a ratio of 1.47 for $e_{\rm v}(1\,{\rm kb})/e_{\rm v}(0\,{\rm kb})$. The experimental ratio for polycarbonate was 1.29 for engineering strains, 1.36 for true strains. Thus, free volume could account for the increased strain to yield.

3.2.4. Behaviour of PTFE at high pressure

The yield stress of PTFE, as defined above, increases with pressure much as do those of the other polymers, up to 4 kb. Then a systematic change is noted. This can be seen in fig. 8. It is known that PTFE has a solid– solid phase transition at about 5.5 kb, judging by the dilatometric data of Weir (1951). Wood (1964) has calculated bulk modulus values at various pressures from these dilatometric results, and his relationship has been superimposed on the present nominal yield stress versus pressure results in fig. 8. There is an obvious similarity in the deviation of yield stress around 4-5 kb and then a likely increase at higher pressures to the behaviour of the bulk modulus. (The behaviour of Young's modulus included in a later discussion, shows a similar deviation.) An exact correspondence is not expected considering the different natures of the phenomena.

3.2.5. Behaviour of Young's Modulus

The values of Young's modulus obtained from the present work were low, not because of an insufficiently stiff system, but because of other experimental considerations. Nevertheless it is still instructive to note the change of modulus as a function of pressure. To this end the relative values have been plotted for the four polymers in fig. 9. Also included are data for POM taken from the work of Sardar *et al.* (1968), using the same experimental conditions as in the present work.



Fig. 8

Engineering yield stress and bulk modulus of polytetrafluoroethylene as a function of pressure. Bulk modulus data after Wood (1964).

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There is no correlation of the pressure dependence of modulus either with the class of polymer or with the yield parameter μ . It is likely that the pressure dependence of the modulus, as with the temperature dependence, depends on the nearness of relaxations and the changes of modulus associated with these, such as in the elastomers studied by Paterson (1964).

The present data do not support the contention that yielding is uniquely associated with a particular ratio of yield stress and modulus. For polycarbonate, the modulus increases by about 40% up to 8 kb whereas the yield stress increases by 240% over the same pressure range. This indicates that the factors controlling the modulus of polymers are not entirely the same as those governing plastic flow. Matsuoka (1965) has briefly touched on the various morphological and structural variables affecting different mechanical properties.

Mechanical Behaviour of Polymers under High Pressure

3.2.6. Fracture strain and the ductile-brittle transition

For PBAC and PCTFE, and likely for PET, the strain to fracture decreases with increasing pressure, the trend pointing toward an eventual fracture before a yield maximum is reached, the usual behaviour of a brittle polymer. This behaviour is in fact qualitatively similar to behaviour with decreasing temperature. Perhaps here more markedly, the relationship between relaxations and changes in large-strain deformation modes may be shown to be of importance, whether the temperature is lowered to reach the relaxation or the pressure is increased to force the transition temperature upward to the test temperature. In this regard the work of Williams (1966) is instructive. His work on the pressure, temperature,





Relative Young's modulus as a function of pressure for five polymers. Date for POM from Sardar *et al.* (1968).

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and frequency dependence of the low-temperature dielection relaxation in PET (near $65^{\circ}c$), which has been associated (Armeniades, Kuriyama, Roe and Baer, 1967) with its ductile-brittle transition, provides the data for extrapolating this change of deformation mode to higher pressures. Extrapolating his data to 1 Hz, it can be replotted as pressure versus the temperature for observation of the frequency maximum, as in fig. 10. Extending the relationship to room temperature ($27^{\circ}c$ here), the transition in behaviour would be expected at pressures near 11 kb, which is above those attained in the present work but in a range that might be anticipated from the present fracture and yield strain results. Correlation of such large-strain mechanical tests with either dielectric or dynamic mechanical tests, all under pressure, would be most useful in pursuing this relationship.





§ 4. CONCLUSIONS

The pressure dependence of the yield stress was fitted using modifications of the common von Mises and Mohr–Coulomb criteria. The parameter representing this pressure dependence showed no discrimination between classes of polymers, and was only found to correlate qualitatively with the strength-limiting temperature, $T_{\rm g}$ or $T_{\rm m}$.

Three polymers, PBAC, PET, and PCTFE, exhibited an increase of yield strain with pressure, and for PABC a calculation based on a free volume concept predicts an increase of yield strain with pressure close to that determined experimentally.

An approximate method for superposing the effects on yield stress of temperature and pressure was developed as a pseudo-thermodynamic equation. The decrease of fracture strains toward an implicit ductile– brittle transition with increasing pressure may be associated with a relaxation whose observation temperature shifts upward with pressure, but which normally is associated at low temperatures with a ductile–brittle transition.

For the materials considered, the relative Young's modulus increased with pressure, the semi-crystalline polymers showing larger changes than the amorphous materials. However, it is felt this may be due to the proximity and magnitude of relaxations in these materials rather than to morphological factors.

Polytetrafluoroethylene behaved uniquely. Although the arbitrarily defined yield stress showed a pressure-dependence similar to the other materials up to 4 kb, above that the yield stress, as well as Young's modulus, showed systematic deviations. Using independent bulk modulus results, these deviations were correlated with a solid-solid phase transition in this pressure range.

APPENDIX

Several results from application of the two yield criteria used above with two common stress states are of general interest and will be developed below. In the following, normal stresses are positive if tensile, pressure P is always a positive quantity, shear stress τ are always positive, and T_{ij} is a deviatoric stress in the j direction caused by a force in the i direction. To discriminate between the criteria but display the results for quick comparison, the equations pertaining to one or the other criteria will be placed on a particular side of a centre line, following the example

Modified von Mises criterion

$$\tau_{\rm oct} = \tau_{\rm s} - \mu T_{\rm m} \qquad \tau_{\rm max} = \tau_0 - \mu' \sigma_{\rm m} \ . \ . \ . \ . \ (5)$$

(I) For a uniaxial test with $\sigma_1 = T_{1\bar{1}}P$ and $\sigma_2 = \sigma_3 = -P$, the

By definition,

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where the upper sign refers to tensile tests and the lower to compression tests. Combining eqn. (6) with eqn. (7),

$$\tau_{\rm oct} = \pm \sqrt{2}(\sigma_{\rm m} + P) \qquad \qquad \tau_{\rm max} = \pm \left(\frac{3}{2}\right)(\sigma_{\rm m} + P)$$

and then imposing the yield criteria, eqns. (3),

$$T_{11}{}^{\rm Y} \!=\! (3/\!\mu \pm \sqrt{2})(\tau_{\rm s} \!+\! \mu P) \qquad \qquad T_{11}{}^{\rm Y} \!=\! [3/\!\mu' \pm (\frac{3}{2})](\tau_0 \!+\! \mu' P)$$

The ratios of compressive-to-tensile true shear-yield stresses would then be

$$\frac{T_{11}^{\rm Y}(C)}{T_{11}^{\rm Y}(T)} = \frac{\mu + \sqrt{2}}{\mu - \sqrt{2}} \qquad \qquad \frac{T_{11}^{\rm Y}(C)}{T_{11}^{\rm Y}(T)} = \frac{\mu' + \left(\frac{3}{2}\right)}{\mu' - \left(\frac{3}{2}\right)}. \qquad (8)$$

These can be rearranged to solve for the parameter describing the pressure dependence,

$$\mu = \sqrt{2} \frac{T_{11}{}^{\mathrm{Y}}(C) + T_{11}{}^{\mathrm{Y}}(T)}{T_{11}{}^{\mathrm{Y}}(C) - T_{11}{}^{\mathrm{Y}}(T)} \qquad \qquad \mu' = (\frac{3}{2}) \frac{T_{11}{}^{\mathrm{Y}}(C) + T_{11}{}^{\mathrm{Y}}(T)}{T_{11}{}^{\mathrm{Y}}(C) - T_{11}{}^{\mathrm{Y}}(T)}$$

which for those uniaxial tests sets,

$$\mu = (2\sqrt{2})\mu'/3.$$

Equating the expressions for $T_{11}{}^{\rm Y}$ of the two yield criteria and substituting from the last equation results in

$$\tau_{\rm s} = (2 - \sqrt{2}) \tau_0 / 3$$

(II) For pure shear biaxial tests,

$$\sigma_{\rm m} = -P$$

and, by definition,

$$au_{
m oct}(-\sqrt{\frac{2}{3}})T_{12}(S)$$
 $au_{
m max} = T_{12}(S)$

Imposing the respective yield criteria, eqns. (3),

$$T_{12}{}^{\rm Y}(S) = (-\sqrt{\frac{3}{2}})(\tau_{\rm s} + \mu P) \qquad T_{12}{}^{\rm Y}(S) = (\tau_0 + \mu' P). \qquad . \qquad . \qquad (9)$$

At atmospheric pressure, where $\sigma_m = 0$, which is the common situation for shear tests,

$$\tau_{\rm s} \!=\! \tau_0 \!-\! \sqrt{(\tfrac{2}{3})}$$

8

Equating the expressions for $T_{12}{}^{Y}(S)$ of the two yield criteria and substituting from the last equation results in

$$\mu = \mu' \sqrt{\left(\frac{2}{3}\right)}$$

The equations all revert to accepted forms at atmospheric pressure $(\sigma_{\rm m}=0)$ or for pressure-independent yield stresses $(\mu=0)$. Note that although μ and $\tau_{\rm s}$ refer to any stress state, μ' and $\tau_{\rm 0}$ vary with the stress state.

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